Forced mixing in boundary layers

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The effect of increasing the rate of mixing in turbulent boundary layers in a region of adverse pressure gradient has been investigated experimentally. Only the two-dimensional case was considered. The boundary layer was formed on a flat wall in a special wind tunnel in which a variety of adverse pressure gradients could be obtained. Speeds were low enough to justify the neglect of compressibility. The main objective was to compare the effect of increasing the rate of mixing with the effect of reducing the pressure gradient on boundary-layer development and separation. A variety of mixing schemes was tried, all of them involving fixed devices arranged in a row on the surface in the region of rising pressure. While these differed considerably in effectiveness, they had a generally similar effect on the flow; and, except for effects arising from changes in displacement and momentum thickness introduced at the devices, their effect on the layer was basically equivalent to that of a decrease in pressure gradient. Apart from forced mixing, the shape of the pressure distribution was found to have a significant effect on displacement and momentum thickness, these being minimized and the wall distance decreased for a given pressure rise by a distribution with an initially steep and progressively decreasing gradient.

1. Introduction

When the relative motion of a fluid near the surface of a body has been reduced by friction or other momentum-extracting agent, there arises a problem of maintaining flow in regions where the pressure is increasing in the flow direction. The well-known phenomena of excessive boundary-layer thickening and flow separation are manifestations of the reduced ability or inability of the flow to proceed to regions of higher pressure.

These phenomena have been studied very extensively as part of the general boundary-layer problem and are sufficiently well understood to provide some rational rules of design by which unwanted effects may be avoided or reduced. In essence these are rules for exploiting natural turbulence, and in general they amount to giving the body a shape and attitude so that the necessary pressure recovery will occur gradually enough to enable natural turbulent mixing to keep the flow from stagnating under the action of the opposing pressure. The designer can, of course, resort to the expedient of partial removal of the boundary layer by suction or to energizing it by blowing through slots.

While the exploitation of natural turbulent mixing is the basis of most design, the possibility of increasing the rate of mixing by auxiliary devices has been explored in a number of technical applications. The general purpose of this investigation is to find the effect of augmented mixing on boundary-layer development and flow separation. The major theme is the comparison between pressure gradient and forced mixing as separate controllable parameters. Ultimately we seek the answer to the following question: What difference is there between applying forced mixing to increase the pressure recovery that a boundary layer will withstand before separation occurs, and the alternative of lowering the pressure gradient to permit natural turbulent mixing to delay separation until the same pressure recovery is achieved?

The idea of using mixing devices to assist flow against an opposing pressure gradient and delay or avoid separation is not new. As far as is known to the authors, the first device of this kind was the vortex generator, devised in 1946 by H. D. Taylor and associates of the Research Department of the United Aircraft Corporation. A summary account of this work including references to ten other reports is given by Taylor (1950). Subsequently other devices have been tried by various workers, and a variety of applications of these and vortex generators have been studied, mostly with emphasis on delaying separation and improving performances of airfoils and diffusers. The following references include the known schemes: Grose (1954); McCullough, Nitzberg & Kelly (1951); Pankhurst (1955); Stephens & Collins (1955); Taylor (1948, 1950); Weiberg & McCullough (1952). No reference to the various applications is attempted. The report by Grose (1954) was discovered when the present investigation was about completed. In it we find some parallel between that work and our own, as both deal with the boundary layer on a flat wall and both explore the effectiveness of various types of mixing devices. There seems, however, to have been no study of the problem with the specific aim of the present investigation.

Since this is a boundary-layer study rather than a performance study of some particular body, a two-dimensional turbulent boundary layer on a smooth flat wall was used. The pressure distribution along the wall is independently controlled and is made to rise from a nearly constant level over the fore part of the wall to higher levels at rates (gradients) sufficient to cause flow separation. This produces the pressure-recovery region, which is the only part of a pressure distribution with which this investigation is concerned. The medium is air at speeds low enough to justify the neglect of compressibility effects, but high enough, in conjunction with linear dimensions, to afford Reynolds numbers of technical interest.

Mixing devices presented a special problem. While no moving or running devices were considered, it was realized that an almost endless variety of fixed devices could be conceived. The best hope was that enough schemes could be tried to give a general idea of what could be expected.

An account of this investigation was originally given in *NBS Rep.* 6107 to the Office of Naval Research. In the present paper new data have been added and all results have been re-evaluated in the light of the more complete information. While it is fully realized that this study covers only a limited part of a complex subject, it is hoped that the information will serve to show how forced mixing fits into the pressure-recovery problem and give some idea of its probable significance.

2. Purpose and nature of mixing

A boundary layer comprises fluid that has insufficient momentum to flow as far up a pressure rise as fluid outside the layer. In the limit at the wall the momentum vanishes and with it the ability of the fluid to make any progress against a pressure rise. Thus we see that flow to higher pressure would cease, first at the wall and then farther out, unless maintained by some mechanism that causes the faster moving fluid to render assistance to the slower fluid. Since there is friction, moving fluid tends to drag along fluid that would tend to stop. However, we know that we must look to something other than internal friction between smoothly flowing layers, which is the well-known friction of viscous origin in laminar flow, if we are to meet practical requirements.

Just as we find in many cases that we must resort to stirring when molecular diffusion alone is inadequate to do the desired mixing, so here we find that stirring, which mixes the slower fluid near the wall with faster fluid farther out, greatly facilitates the movement of the less energetic fluid. Just such mixing is provided by nature in turbulent flow. The effect is often expressed as a turbulent shear stress, and by analogy with molecular processes, the agency responsible is likened to a viscosity and is called 'eddy viscosity'. When we consider the fact that eddy viscosity in boundary layers is of the order of 100 times greater than ordinary viscosity, and in jets and wakes is of the order of 1000 times, we see that turbulent flow embodies a relatively powerful self-mixing mechanism. The numbers given are only order-of-magnitude ones, as the actual value of the ratio is proportional to the Reynolds number. This powerful self-mixing makes it possible for turbulent flow to negotiate much steeper and higher pressure rises than can laminar flow. However, even with such capabilities the upper limit is often inconveniently low, and we have reason to expect that we can benefit by a mixing more vigorous than that presented to us naturally, if this is to be had without at the same time introducing other factors that may nullify the possible gain. The benefits consist of delaying or preventing stagnation and the resulting flow separation from a wall and in generally broadening our capabilities for achieving pressure recovery.

We may examine the effect of mixing on the mean flow field without becoming involved with the detailed flow processes by examining the effect on the integral parameters δ^* and θ and the shape parameter H, where

$$\delta^* = \text{displacement thickness} = \int_0^\delta \left(1-\frac{U}{U_1}\right) dy,$$

$$\theta = \text{momentum thickness} = \int_0^\delta \left(1-\frac{U}{U_1}\right)\frac{U}{U_1}\, dy,$$

$$H = \delta^*/\theta.$$

Here y is distance from the wall; U is the velocity at any point in the boundary layer; U_1 is the free-stream velocity outside the layer; and δ is the ordinary boundary-layer thickness, taken as the value of y where U is not measurably different from U_1.

A useful relationship in this connexion is the von Kármán momentum integral equation, written as follows for two-dimensional flow

$$\frac{d\theta}{dx} = \frac{\theta}{q}\frac{dp}{dx}\left(\frac{1}{2}\frac{\delta^*}{\theta}+1\right)+\frac{\tau_w}{2q}. \tag{1}$$

where x is distance along the wall; p is the static pressure, assumed constant across the boundary layer; q is the free-stream dynamic pressure; and τ_w is the shear stress at the wall. As is customary, the term expressing the Reynolds normal stress in turbulent flow has been omitted. The two-dimensional form is used because we are concerned only with a two-dimensional boundary layer in this investigation. While stationary three-dimensional flow patterns will be encountered when forced mixing is applied, it will be assumed that equation (1) is applicable when an average is taken over enough spanwise-distributed patterns to yield representative average values of δ^* and θ.

Our purpose now is to see in terms of δ^* and θ how the boundary layer is affected by mixing. Equation (1) is an expression for the growth of θ with x in terms of the forces exerted on the flow. These forces arise from the pressure gradient and wall friction and are expressed in dimensionless form by the first and second terms, respectively, on the right-hand side of the equation. The retardation of the flow by the forces causes δ^* to increase also. However, the value of δ^* depends on the amount of fluid through which the retardation is distributed, as is perhaps best illustrated by the well-known change in H which occurs with transition from laminar to turbulent flow. With the introduction of turbulence an increase in mixing takes place in a distance so short that θ remains virtually unchanged. For the case of zero pressure gradient, H goes from about 2·6 in the laminar layer to 1·3 in the turbulent layer. This means that δ^* is decreased to half of its original value by the mixing action of turbulence.

It is seen therefore that the more the momentum loss is dispersed throughout the fluid, the less is the flow displacement caused by the retarding forces. Mixing therefore affects δ^* directly. As shown by equation (1), mixing can affect the growth of θ through its explicit effect on the value of δ^*. The effect is to decrease $d\theta/dx$ unless this is offset by an increase in τ_w. Since δ^* occurs in the ratio δ^*/θ, it is customary to discuss its effect in terms of the shape parameter, H. The determination of H is in fact the major task of the several methods that have been proposed for calculating turbulent boundary-layer development.

The quantity τ_w is itself reduced by an adverse pressure gradient, and if the gradient is high enough, the τ_w-term of equation (1) may become negligible compared with the dp/dx-term. The special case where $\tau_w = 0$ and $H =$ const. is of some interest, and for this case the equation can be integrated directly to give

$$\frac{\theta}{\theta_i} = \left(\frac{q_i}{q}\right)^{(H/2+1)} \left\{\begin{matrix} H = \text{const.} \\ \tau_w = 0, \end{matrix}\right\} \tag{2}$$

where θ_i and q_i are initial values. The skin friction τ_w will not be zero unless the flow is always in a state of incipient separation, and equation (2) becomes a justifiable approximation only when the τ_w-term is small compared with the dp/dx-term in equation (1). The other condition, $H =$ const., will not in general be met, but this will not affect the physical insight afforded by equation (2). Since the skin friction does not enter into the equation, it expresses the growth of θ resulting purely from carrying boundary-layer fluid from a dynamic pressure of q_i to q. This is the only aspect of the growth that mixing can affect favourably, and the effect comes about from a reduction of H. It is known that H is also reduced by reducing the pressure gradient; hence we expect reduced pressure gradient and increased rate of mixing to have similar effects. The similarity may be explained on the grounds that forced mixing and reduced pressure gradient are merely means of assisting mixing to expedite the flow—the one by increasing the rate and the other by increasing the time available. Accordingly, if H is to be held at some fixed value, an increased rate of mixing will make possible a larger pressure gradient. Again, if H is increasing with x, an increase in the rate of mixing will decrease dH/dx, or hold dH/dx the same for some larger value of pressure gradient. Since the separation condition is associated with a particular value of H ($H \simeq 2$), an increase in the rate of mixing can be expected to make the pressure recovery higher before separation occurs, particularly where the pressure gradient is large. Only qualitative estimates of this kind are possible because any reduction of H below its separation value normally brings skin friction into the picture.

We must assume that stirring by any means will increase skin friction by virtue of bringing higher velocities nearer the wall. In this connexion an interesting observation may be made. When mixing takes place by eddy motions comprising bulk movements of fluid, as it does in turbulent mixing, the mixing weakens as the wall is approached and finally disappears leaving only molecular viscosity in a laminar sublayer at the wall. Thus, while the skin friction has been increased, this has come about because of a velocity increase near the wall rather than by a uniform, across-the-layer increase in eddy viscosity. This carries with it a compensating feature, namely, a shaping of the velocity profile toward lower values of H. For this reason values of H in turbulent flow are lower than those in laminar flow. For example, the values of H computed by Clauser (1954) for equilibrium laminar profiles range from 2·592 for constant-pressure flow to 4·031 for a pressure gradient sufficient to maintain a state of incipient separation. For comparison the corresponding values of H for turbulent flow range from about 1·3 to 2. Thus, with turbulence setting the pattern, a good case can be made for mixing, and we are afforded ample justification for attempting to increase it. Care should be exercised that the advantage is not lost through the scheme of mixing adopted. We know, for example, that the advantage would be lost by increasing the rate of mixing by roughening the wall.

When we begin to consider how to go about mixing, we are at once struck by the fact that nature's scheme of turbulent mixing approaches the ideal one. It is simply a self-induced mixing arising out of the velocity differences already existing. The forces are applied by one portion of the fluid on another, and no momentum loss is charged against the mixing process. If we are to match this ideal, we should not consume momentum of mean flow to do the mixing, but should rearrange the velocity field so as to produce steeper gradients within which turbulent mixing motions are generated and across which they transfer momentum. A pure rearrangement implies guiding the fluid to new positions, leaving no residual motions to require an induced drag from the mixing devices. No fluid-handling device can, however, escape the drag that arises from frictional effects; hence if residual motions contribute enough to the mixing, suppression of them would not be warranted. The well-known vortex generator is an example wherein the derived mixing depends on induced, streamwise vortices. In fact such vortices may be regarded as a rearranging mechanism installed in the flow and persisting for a considerable distance downstream. The study of mixing devices is taken up in § 5.

3. Experimental arrangement

A wind tunnel built specifically for this investigation is shown in figure 1. The boundary-layer studies were conducted on the bottom wall of the test section. This wall was a smooth flat surface 6 ft. wide made of birch plywood. The top wall consisted partly of slatted sections for the purpose of producing a static pressure increase by progressively reducing the volume flow in the duct. By adjusting the slats the boundary layer on the bottom wall could be subjected to various adverse pressure gradients. This type of control was made possible by supplying air from the room to the system under positive pressures by the arrangement shown. The screened diffuser on the outlet of a centrifugal fan and the settling chamber containing a honeycomb and additional screens made the flow entering the test section sufficiently uniform and free from turbulence for the job at hand.

The test section was originally $13\frac{1}{2}$ in. deep and 16 ft. long, but during the course of the investigation it became necessary to extend the length to 24 ft. with the slope of the top adjustable to a final depth of 27 in. The boundary layer was tripped about 1 ft. from the entrance to produce a definite beginning of the turbulent layer. The region of pressure rise was begun after an initial 4 ft. run of constant pressure. All reference quantities pertain to conditions at the 4 ft. position, and this position is taken as the origin of x. Here the boundary layer was about 1 in. thick. The free-stream velocity at the 4 ft. position was maintained at 82 ft./sec, except when altered to investigate scale effects. The corresponding Reynolds number, based on the 4 ft. distance from the approximate beginning of the layer, was $1{\cdot}9 \times 10^6$.

Separate total-head and static-pressure tubes, made from 0·04 in. outside diameter nickel tubing, were used for the measurement of velocities and pressures throughout the boundary layer. Gear for traversing in all directions was provided. By flattening the end of the total head tube to an opening of 0·005 in. it was possible to make reliable measurements sufficiently near the wall. Pressures at the wall were measured by means of $\frac{1}{16}$ in. orifices in the wall itself. To check the spanwise uniformity these were placed in three rows running the full length of the wall, one on the centreline and the other two 15 in. to each side.

A dust method, developed earlier in the laboratory, was used to indicate flow separation. This consisted simply of injecting talc dust into the region downstream of separation and permitting the currents in the wake to carry the dust upstream to the line of separation. A film obtained by wiping the surface with an oily cloth was sufficient to hold the dust and produce a clearly defined boundary. A typical example showing the region of separation downstream from a row of mixing devices is shown in figure 2. The white line further upstream shows where the line of separation was with devices absent. Since the lines of separation were never straight, an average was taken over the central 45 in. in order to define a mean position. Since the position fluctuates up and downstream, the dust method gives an upstream extreme. Additional examples of the application of this method are given by Smith & Murphy (1955).

An important additional piece of equipment was an especially designed balance installed beneath the test wall for measuring the drag of individual mixing devices when in position in the boundary layer. The mixing device was connected to the balance by a spindle running through a hole in the wall. The balance was sealed to prevent leakage through the hole.

4. Pressure gradients and effec on boundary layer

In order to establish a basis for later investigating the effect of forced mixing, the boundary layer was studied with various adverse pressure distributions which were sufficient in each case to cause eventual separation of the layer. A considerable amount of effort went into attaining a two-dimensional flow. The test for two-dimensionality was the agreement between boundary-layer profiles measured on the centre line and those measured 15 in. to each side. The usual tests were applied to the profiles themselves, namely, agreement with the law of the wall, best agreement being found using the coefficients given by Coles (1956), and agreement with the family of H-parameter profiles given by von Doenhoff & Tetervin (1943). For the lower pressure gradients there were departures from the von Doenhoff–Tetervin set like those found by Clauser (1954) for his equilibrium profiles. Since the form of the profile indicated agreement with the law of the wall, the method suggested by Clauser for obtaining the local skin-friction coefficient from the velocity profile was used.

There was a measureable pressure gradient across the boundary layer, but this was considered to be sufficiently small to permit our taking the pressure at the wall as the pressure applicable to the entire cross-section of the layer. In the determination of velocities, however, the local static pressure was always used.

The several pressure distributions, labelled A to F, are shown in figure 3. In all cases the pressure has been held constant up to the 4 ft. position, where the origin of x is taken, and then made to rise as shown in the figure. The amount of rise is termed the pressure recovery, and this is expressed in terms of a pressure-recovery coefficient $(p-p_0)/q_0$, where p_0 and p are pressures at the wall and q_0 is the free-stream dynamic pressure at the position $x = 0$. Distribution A was set up first in order to provide a condition of early separation and low-pressure recovery to which forced mixing could be applied with some expectation of improvement. Examples of the effect of mixing devices are shown in figure 7. Distribution B and C were next established in order to match as closely as possible the range of pressure recoveries obtained with mixing devices applied to condition A. On the assumption that skin friction would play a minor role in

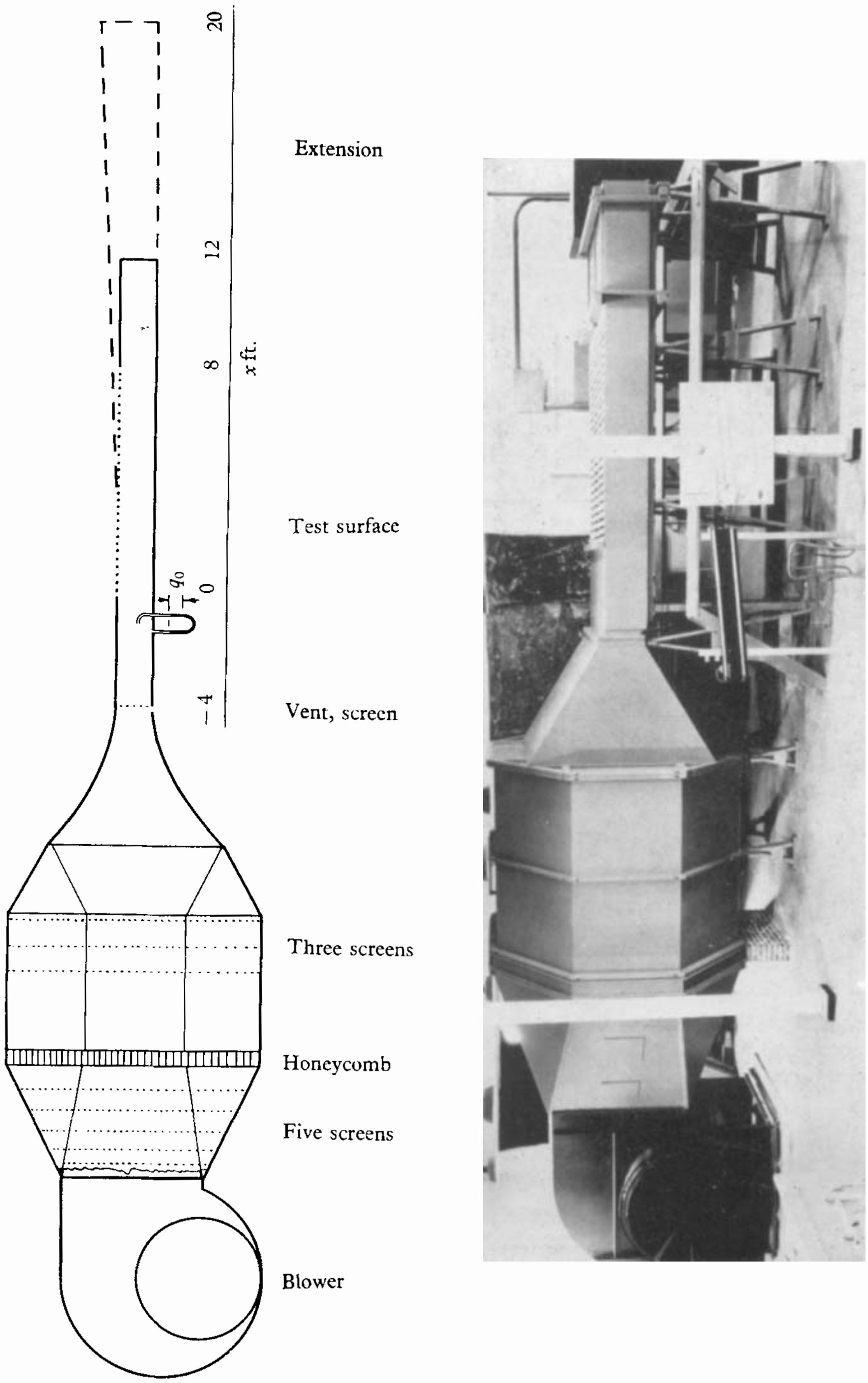

FIGURE 1. Wind tunnel for boundary-layer investigation.

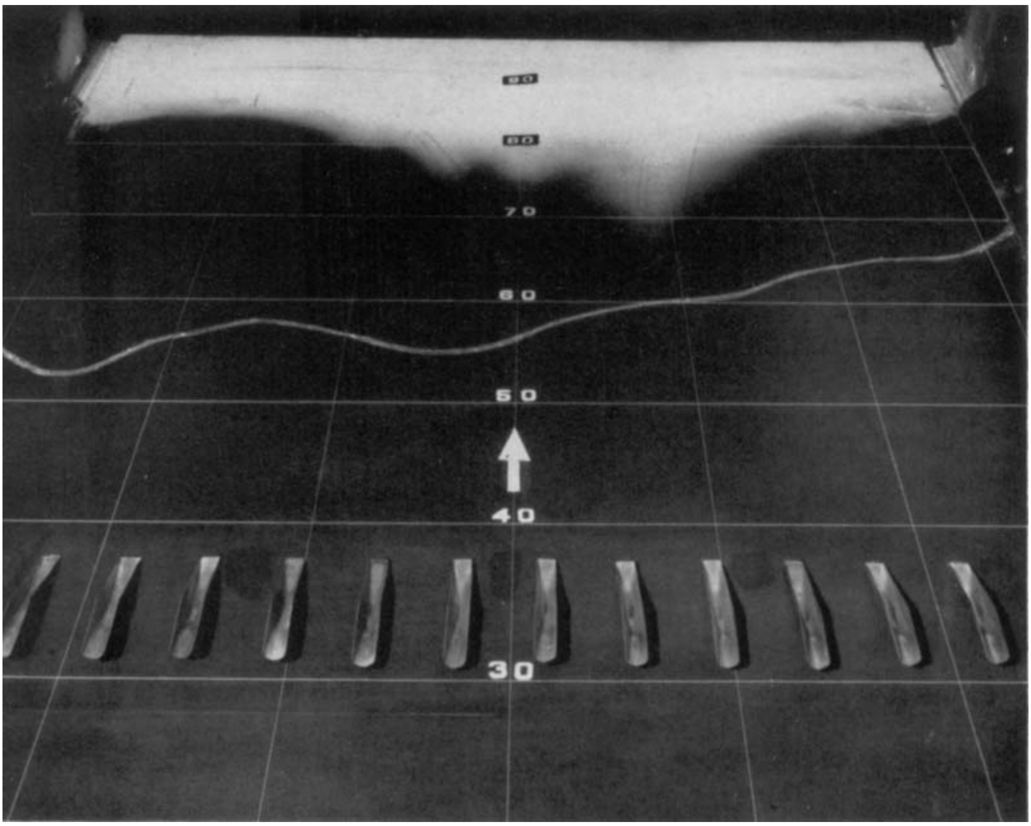

FIGURE 2. Photographs of dust pattern on surface showing separation region (white). The line marks the boundary found in a previous test with mixing devices absent (the dust is now removed). Numbers are distances in inches from the beginning of the pressure rise. The scoop type of mixing device is shown (see figure 6).

boundary-layer development, the idea behind B and C was simply that of promoting flow to higher pressures by reducing the gradient. After the main part of the investigation had been completed, it was suspected that skin friction might be more important than originally supposed, and the investigation was extended to include distributions D, E and F. Forced mixing was applied only in the case of distribution A.

This somewhat elaborate study of naturally turbulent boundary layers was occasioned by the desire to compare the effect of forced mixing to that of the pressure field alone. When the first comparisons were made using only B and C, it was assumed that the shape of the pressure distribution could be ignored and that the rate of pressure rise was the really important quantity. However, the

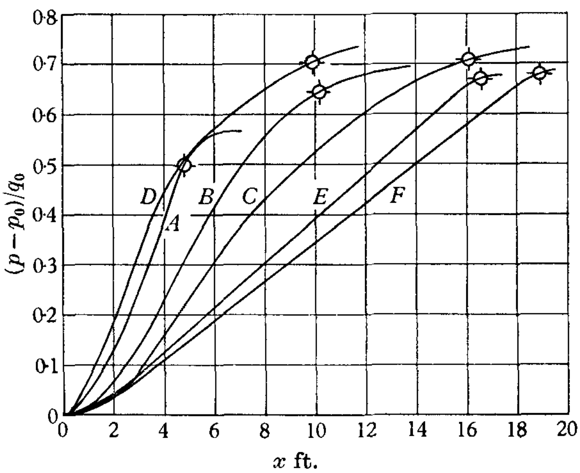

FIGURE 3. Pressure distributions. The origin of x is taken 4 ft. from the leading edge. The pressure is uniform for $-4 \leqslant x \leqslant 0$. ⌽, Separation point.

extended study showed a considerable dependence on shape and indicated the necessity for specifying the shape before a firm basis of comparison would be possible. This is apparent in part from the curves of figure 3. Distributions C and D, for example, achieve closely the same pressure recovery, yet C requires an additional 6·3 ft. run of wall.

A more important aspect of the dependence on shape is examined in figure 4. This type of plotting, in which δ^*, θ, and H are plotted against q_0/q, has been found useful for comparing boundary-layer development under different conditions. It amounts to expressing the thickness parameters as functions of pressure rise, now expressed in terms of q_0/q in order to obtain a more open scale than that afforded by $(p-p_0)/q_0$. The connexion is $q/q_0 = 1-(p-p_0)/q_0$. The diagram is most useful in comparing cases where pressure recoveries up to separation are closely the same. Taking C and D as fitting examples, we see that the boundary layer remains thinner for condition D than for C. The curve for condition E has been added to show that the trend is toward greater values of δ^* and θ as the mean pressure gradient is reduced. The opposite trend is exhibited by the H-curves, and the effect of this, as shown by equation (2), is to oppose the observed trend of θ. In other words, since lowering the pressure gradient reduces H, it should likewise reduce θ and also δ^*. Failure to do so is evidence of an effect of skin friction.

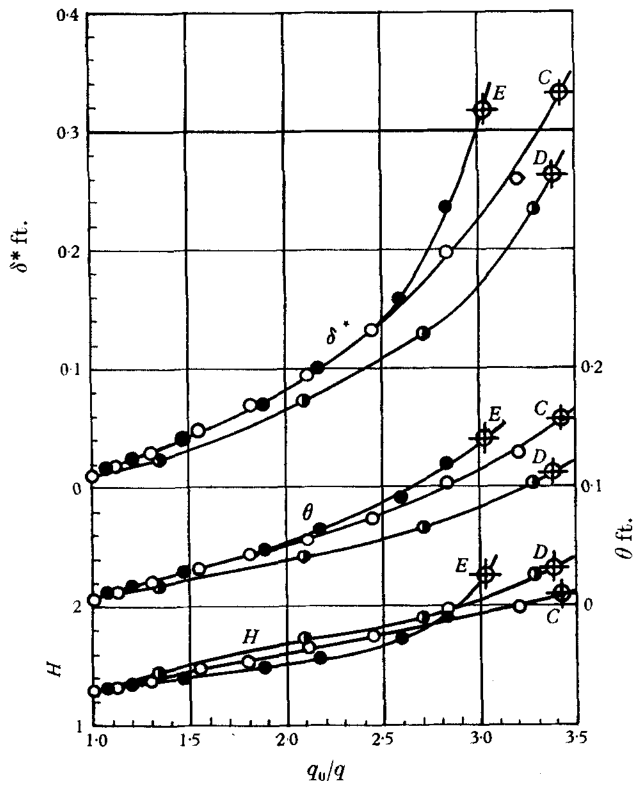

FIGURE 4. The effect of pressure distribution on the relation between δ^*, θ, H. The pressure rise is expressed as q_0/q, where $q/q_0 = 1-(p-p_0)/q_0$. Pressure distribution: C, ○; D, ◑; E, ●. ⊕ Separation point.

The values of the local skin-friction coefficient, τ_w/q, for the naturally turbulent layer were found to decrease monotonically from around 0·0032 at $x = 0$ to around 0·0003 at the indicated separation point. Failure to reach zero is attributed to the fact that the dust method indicates the upstream extreme of a fluctuating separation point. The question of immediate interest is the relative effect of pressure gradient and skin friction on the growth of θ. In this connexion figure 5 has been prepared showing the ratio of the pressure-gradient term to the skin-friction term appearing in equation (1). We see that ultimately the effect of skin friction becomes small by comparison, but when the pressure gradient is low there is a considerable range of x over which it is not negligible. The advantage of distribution D over B, C, E and F arises from the fact that the initially steep pressure gradient of D reduces the skin friction early in the course of the pressure climb and at the same time reduces the length of wall over which the flow proceeds in attaining a given pressure recovery. These effects outweigh the adverse effect arising from the early increase in H shown in figure 4. Comparing

D with A, we find that whereas separation occurs early in A it has been delayed in D by the progressively decreased gradient.

It appears from this that the best form of pressure rise is one that has an initially steep and steadily decreasing gradient. It has been proposed by Stratford (1959) that the optimum pressure curve should be one with gradients such that the flow is on the verge of separation and the skin friction is zero

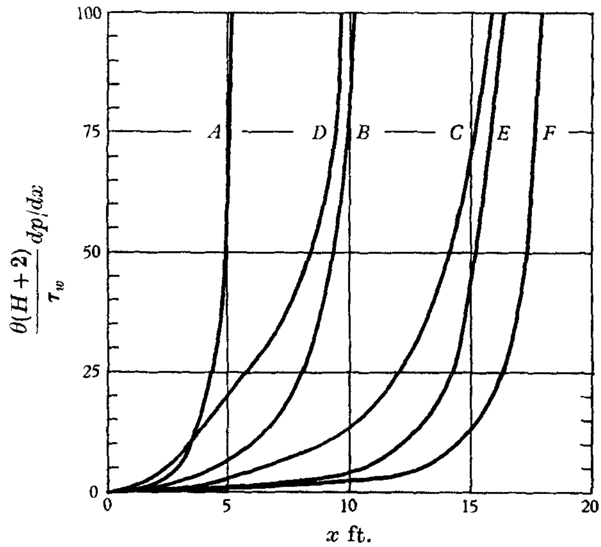

FIGURE 5. Ratio of pressure-gradient term to skin-friction term (1st term to 2nd term on right-hand side of equation (1)) as a function of x for the pressure distributions of figure 3.

throughout. Stratford found the flow to be stable under this condition for his experimental arrangement. In the present investigation, however, it was observed that distribution D was about the limiting high-gradient condition for a stable flow.

5. Mixing devices and effect on separation

While the goal in mixing was reasonably clear, the question of how to accomplish it in a practical manner was quite another matter. The idea, mentioned earlier, of promoting self mixing by simply rearranging the flow was adhered to, but it was found after considerable effort that forms designed to minimize induced drag by cancelling off residual vortex motions generally has as much or more drag than those that produced strong vortex trails. Furthermore, they were always less effective, showing that a vortex with its axis along the stream comprised a sustained rearranging mechanism. It was also learned that mixing on a coarse scale by relatively large, widely spaced devices was far more effective than fine scale mixing; and under these conditions, multiple rows were less effective than a single row of devices properly spaced and properly stationed. Attention was therefore centred on the single-row arrangement as illustrated in figure 2. The devices so studied are shown in figure 6. All except device (B) were produced in sufficient number to span the 6 ft. width of wall and tested for their effect on separation and pressure recovery with the channel arranged for pressure distribution A. The spacing and position for maximum delay of separation and maximum pressure recovery were found by trial and error. A height of the order of the boundary-layer thickness was adopted, but this was not varied in the process of finding the optimum position. The drag of the device was measured

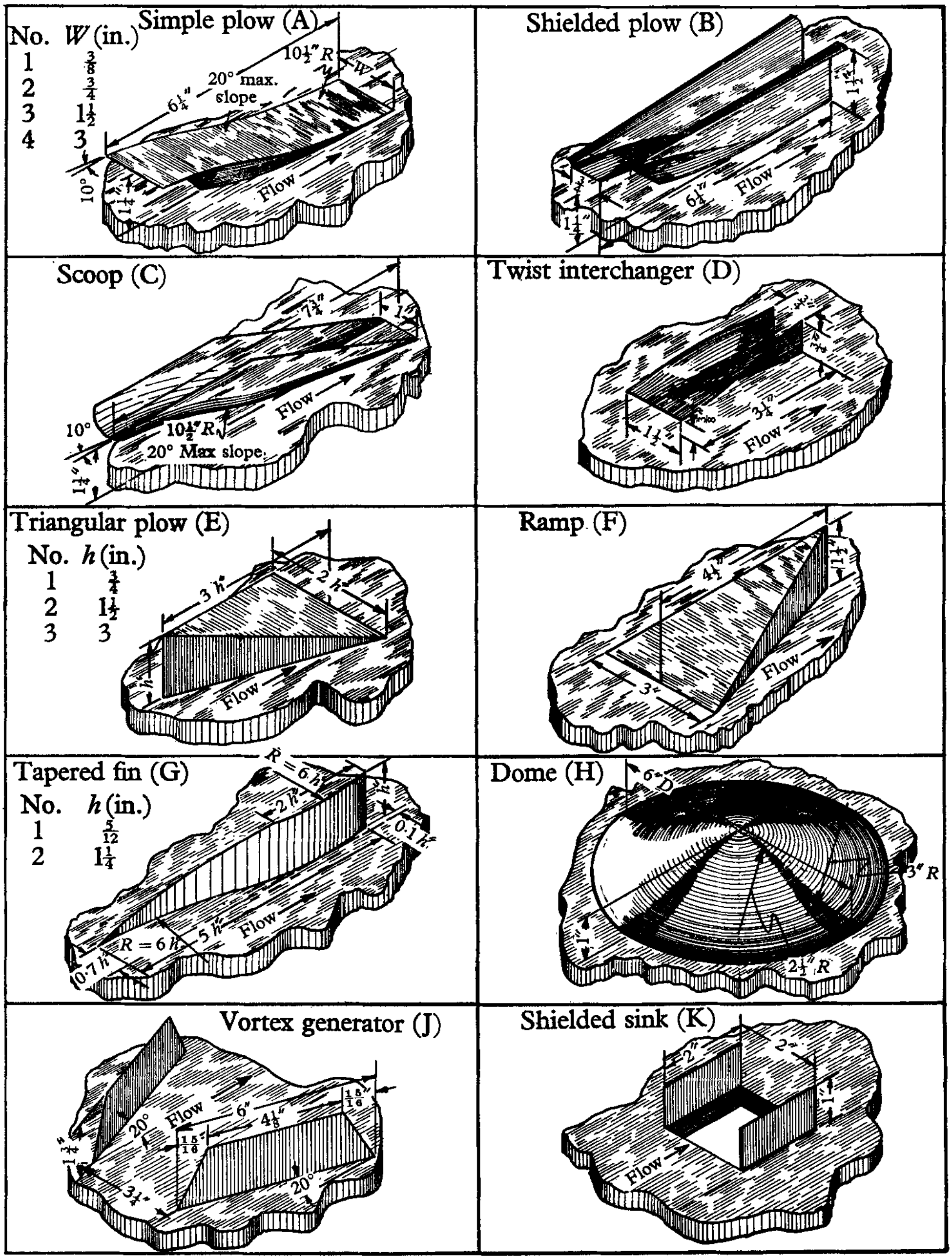

Figure 6. Mixing devices.

in its position in a row or with an identical device to each side. No rotating devices or moving agitators were considered.

Before taking up the results, the reasoning behind the devices and their intended action will be discussed. Device (A), called the simple plow, was intended for parting the boundary layer and guiding outer flow toward the wall into the furrow so produced. The motion thus started continued downstream in the form of a pair of trailing vortices. The shielded plow (B) is device (A-2) modified by the addition of enough side shielding to effectively eliminate the vortices. Since the elaborate additions merely increased the drag of the device,

No.	Device Name	L (in.)	x_t (ft.)	x_s (ft.)	$\frac{p_s-p_0}{q_0}$	$\frac{\Delta p_s}{q_0}$	$\frac{D}{q_t}$ (ft.²)	$\Delta\theta_t$ (ft.)	$\frac{\Delta\theta_t}{\theta_t}$
	None	—	—	4·83	0·50	0	0	0	0
A-1	Simple plow	6	0·52	6·33	0·61	0·11	0·0010	0·0010	0·112
A-2	Simple plow	4·5	2·60	7·00	0·71	0·21	0·0015	0·0020	0·126
A-3	Simple plow	8	2·60	7·25	0·71	0·21	0·0045	0·0034	0·214
A-4	Simple plow	8	3·85	6·83	0·71	0·21	0·0100	0·0075	0·274
B	Shielded plow	—	0·52	—	—	—	0·0025	—	—
C	Scoop	6	1·48	6·67	0·65	0·15	0·0023	0·0023	0·200
D	Twist interchanger	3	1·94	5·75	0·58	0·08	0·0016	0·0032	0·242
E-1	Triangular plow	6	1·02	6·42	0·66	0·16	0·0015	0·0015	0·147
E-2	Triangular plow	6	2·87	6·75	0·67	0·17	0·0050	0·0050	0·289
E-3	Triangular plow	6	4·08	7·92	0·75	0·25	0·0240	0·0240	0·780
F	Ramp	6†	2·87	5·75	0·59	0·09	0·0037	0·0037	0·214
G-1	Tapered fin	2	0·17	6·42	0·52	0·02	—	—	—
G-2	Tapered fin	4	0·52	6·00	0·62	0·12	0·0033	0·0050	0·562
H	Dome	6	0·50	5·92	0·60	0·10	0·0032	0·0032	0·364
J	Vortex generator	6·75	3·80	7·17	0·69	0·19	0·0097	0·0086	0·323
K	Shielded sink	6‡	2·67	6·75	0·67	0·17	—	−0·0030	−0·176

L and x_t are optimum spacing and position, respectively.
x_s is the mean position of separation over the central 45 in. of span.

$$\frac{\Delta p_s}{q_0} = \frac{p_s - p_0}{q_0} - 0{\cdot}50.$$

† The optimum value of L was 3 in. giving $x_s = 6{\cdot}08$ ft. The value 6 in. was used because of interest in comparing devices (F) and (E-2).

‡ Optimum L and x_t not determined.

TABLE 1. Summary of mixer characteristics

this arrangement was abandoned in favour of (C), called the scoop, which incorporated some shielding and was easier to construct. The twist interchanger (D) and the tapered fin (G) represent other ideas for overturning the flow without intentionally involving residual vortex motions. On the basis of the evidence in table 1 it was concluded that drag reduction by elimination of trailing vortices was not practically possible, and since this amounted to stopping the action started at the device, its effectiveness was reduced without compensating benefits. The triangular plow (E) is a less refined version of (A) having much the same action, and the ramp (F) is simply (E) used in reverse. A device nearly like the triangular plow, called a 'wedge-type vortex generator', was included among the devices studied by Grose (1954). The ramp, also called a 'wedge' in the literature, has seen use as a boundary-layer control device. The dome (H) was intended as an object that would add little wetted area and at the same time generate vortices by virtue of its being in the non-uniform velocity field of the boundary layer. Device (J) is a conventional vortex generator of the flat-plate type with a trapezoidal shape approximating the taper recommended by

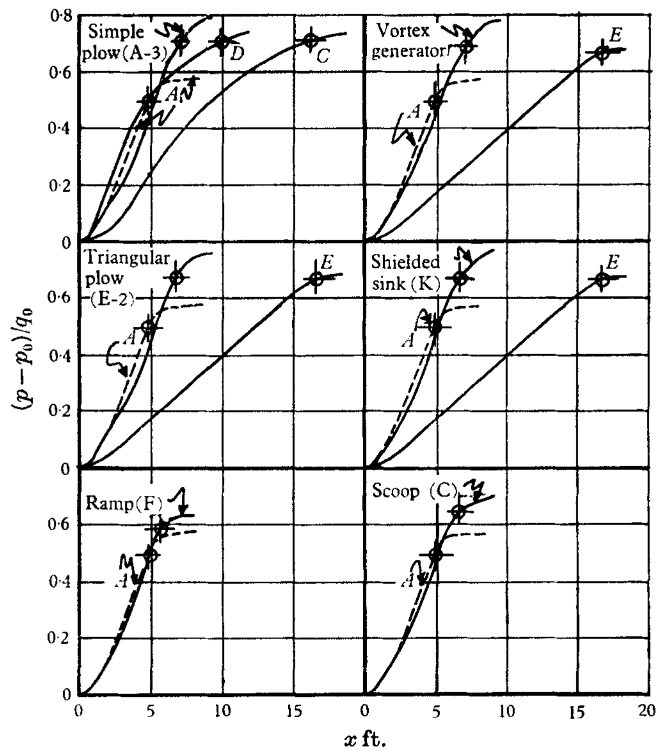

FIGURE 7. Pressure distributions showing typical effect of mixing devices. Distributions A, C, D, E are the same as those given in figure 3. ⊕, Separation point.

Taylor (1948). It was arranged in pairs set for producing counter-rotating vortices. The shielded sink (K) falls in a separate category and will be discussed following the discussion of table 1 and figure 7.

The results are summarized in table 1, and selected examples showing the effect in terms of pressure distribution are shown in figure 7. Curve A represents the condition before mixing devices were applied. The devices extend the curve upward and somewhat modify its earlier course, with the result that there is now a new pressure distribution for each type of device. Curves C, D and E show the pressure distributions designed to produce the same pressure recovery as that obtained by means of the mixing device.

The relative performance of the various devices may be judged from table 1. Starting from a separation point at 4·83 ft. and a pressure-recovery coefficient at separation of 0·5 for no mixing devices, the effect of several devices may be judged by running down columns headed x_s, $(p_s-p_0)/q_0$, and $\Delta p_s/q_0$, where

$\Delta p_s/q_0$ is the increase in the coefficient due to the device. From the drag of the individual members of a set of devices the increment in momentum thickness effectively occurring at the trailing edge of a row of devices was calculated according to the formula

$$\Delta\theta_t = \frac{D}{2Lq_t},$$

where D is the drag of one member, L is the cross-stream spacing between devices, and q_t is the free-stream dynamic pressure at the trailing edge of the device. The fractional increase has been expressed by $\Delta\theta_t/\theta_t$, where θ_t is the momentum thickness at the same position in the absence of mixing devices. The quantity $\Delta p_s/q_0$ may be regarded as a measure of accomplishment, while $\Delta\theta_t/\theta_t$ may be regarded as the price paid for the accomplishment. The price has a special significance which will be considered in § 6.

The shielded sink (K) represents a departure from the usual scheme in that suction is employed. It was envisioned that if air was withdrawn through a hole in the wall the air above that entering the hole would be deflected toward the wall producing an effect not unlike that of a plow, where now the boundary-layer flow passes into the hole instead of being pushed aside. The primary aim was to obtain mixing with a decrement in momentum thickness rather than the increment which characterized external devices. Two-inch round holes spaced 6 in. apart were tried, but it turned out that the mixing action was feeble due to the fact that air was drawn off near the wall for a considerable area around the hole without producing steep cross-stream gradients. The shielded arrangement shown in figure 6 was then tried in the location around the optimum for the other devices. Conditions may therefore not have been optimum for this arrangement. The shields were made one-half hole width high to simulate doors which in practice might be closed to return to a plane wall. The suction quantity was fixed at 5·6 % of the boundary-layer flow, producing the decrement, $-\Delta\theta_t$, shown in table 1. It is seen that the effect on separation point and pressure recovery places the shielded sink with the more effective devices. In order to determine how much of the effect was due to boundary-layer removal alone, the same suction quantity was withdrawn through a continuous slot in the same position. The results compare as follows:

	x_s (ft.)	$\dfrac{p_s-p_0}{q_0}$	$\dfrac{\Delta p_s}{q_0}$	$\dfrac{\Delta\theta_t}{\theta_t}$
Shielded sink	6·75	0·67	0·17	−0·176
Continuous slot	5·17	0·57	0·07	−0·183
None (table 1)	4·83	0·50	0	0

While the continuous slot produced the greater reduction in momentum thickness, its effect on the separation point and pressure recovery was small. This shows that the principal effect of the shielded sink was derived from mixing. In this case, furthermore, the mixing involves a negative $\Delta\theta_t$, the decrement of flow of momentum represented by $-\Delta\theta_t$ being contained in the extracted air. Any additional momentum loss resulting from the use of the device will be added to this, but since it is now internal, it will not result in as great an overall loss as will an equal amount deposited in the boundary layer at the mixer station. The reason for this and the benefits of a $-\Delta\theta_t$ will become apparent in § 6.

6. Effect of mixing on boundary-layer development

Detailed velocity surveys were made throughout the boundary layer with selected sets of mixing devices installed in the positions designated in table 1. Those chosen for this purpose were A-3, C, E-2, F, J and K. The purpose was to find out what was happening to the flow behind such devices and in particular

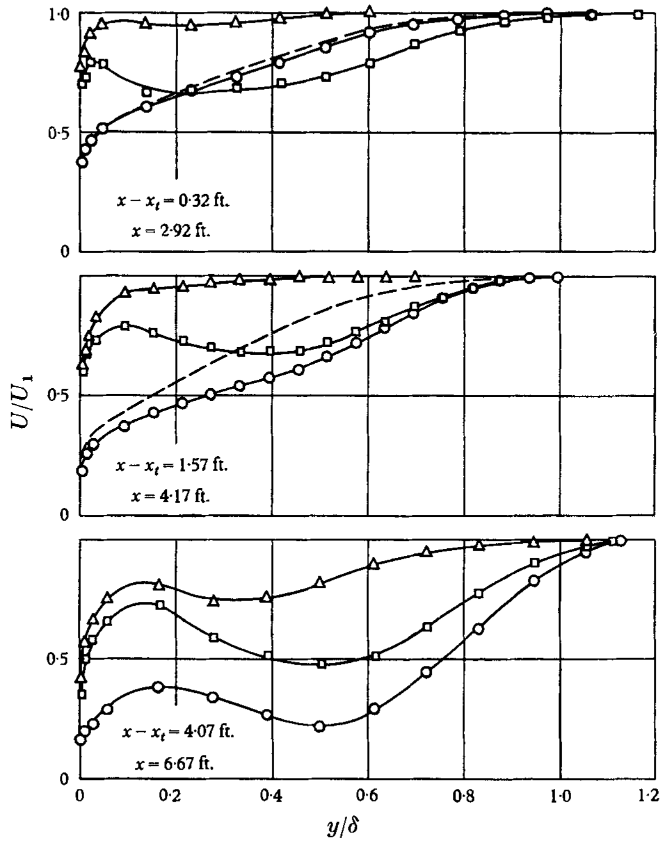

FIGURE 8. Mean-velocity profiles aft of simple plow (A-3) for conditions as given in table 1. Here δ is the boundary-layer thickness at the same station (x) with mixing devices absent. z/L: $\triangle$, 0·00; $\square$, 0·25; $\bigcirc$, 0·50; – – –, without devices.

to ascertain the average state of the flow in terms of an average δ^* and an average θ, the average being taken across the span. The ultimate purpose was to compare the course of development of such average values of δ^*, θ, and their ratio H with the same quantities in the naturally developing boundary layer. The procedure was to traverse in the y-direction, measuring total head and static pressure, at various cross-stream positions. This was then repeated at various distances downstream, and in this way the flow field was completely mapped out.

There were two sources of error not ordinarily encountered in boundary-layer measurements. One arose from strong vortex motions near the trailing edge of a device, and the other arose from erratic cross-stream movements of the flow pattern. It was the uncertainties introduced by the former that prompted the use of drag measurements for finding the $\Delta\theta_t$'s of table 1. However, values derived from velocity traverses agreed reasonably well with those derived from drag measurements. The second source of error caused uncertainties by introducing a considerable scatter in the data, particularly when the velocity gradients in the z-direction were steep. In general the accuracy approached that ordinarily obtained in boundary-layer measurements with pitot-static tubes uncorrected for the effect of turbulence.

An example of a set of velocity profiles is given in figure 8 at various cross-stream positions and various distances behind the simple plow (A-3). Here δ is the boundary-layer thickness for the same station in the absence of mixing devices, the profiles for this condition being shown by the dashed curves in two cases. This example is typical, with variations in degree, of the velocity patterns of the other devices, with the exception of the ramp (F) which showed a generally similar pattern, but with the higher flow rate displaced to the region between devices. This gave rise to alternate rows of high and low rates of flow in the boundary layer which decreased with distance but generally persisted to the separation point. The effectiveness of a device could be correlated with the intensity of the differences and to their persistence. From profiles like those of figure 8, local values of δ^* and θ were derived, and the average of these was then taken to obtain parameters pertaining to an equivalent two-dimensional boundary layer.

From the results obtained with the six devices certain common behaviour patterns emerged, and we shall here concern ourselves with the generalities derived from them. In all cases the operating conditions were as specified in table 1.

The typical change in boundary-layer development brought about by adding mixing devices is shown in figure 9. Not all devices were as effective as the simple plow (A-3), used here as the sample, but performances were generally similar. The drag of the device results in a step-up in δ^* and θ, placing the curves above the dashed curves pertaining to the boundary layer in its original condition. Subsequent increases are slower, and the solid curves approach or go below the dashed curves and then continue with the characteristic steepening rise to the delayed separation point. In the case of the shielded sink (K), where $\Delta\theta_t$ was negative, the step in δ^* and θ was down, and the curves lay below the dashed curves throughout. The values of H were reduced in the typical fashion shown, but again reached about the same value at separation as that found for the natural layer. On the average this value was 2, although it ranged from 1·75 to 2·3. These somewhat lower values than those usually quoted in the literature are attributed to the fact that they apply to the upstream extreme of a fluctuating separation point.

We next take up the comparisons toward which this investigation has been aimed, namely, comparisons between the boundary-layer development when forced mixing was used to attain a given pressure recovery and when the recovery was alternately attained by adjusting the pressure distribution alone. Examples of the performance of three of the devices, illustrated by figures 10 and 11, will suffice to show what can be concluded from such comparisons.

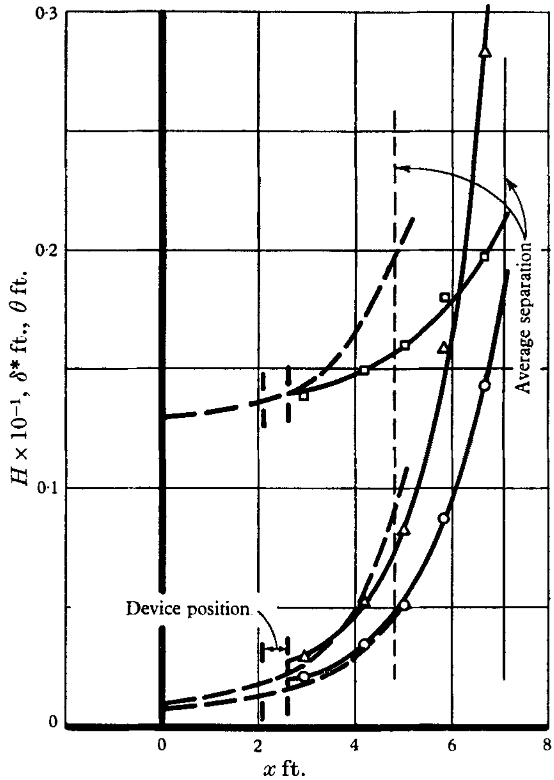

FIGURE 9. Effect of forced mixing on separation and averaged thickness and shape parameters. Here δ^* and θ are mean values derived by averaging local values over a sufficient span to be equivalent to a two-dimensional case. —, Simple plow (A-3); – –, without devices. △, δ^*; ○, θ; □, $H = \delta^*/\theta$.

In figure 10, again illustrating the performance of the simple plow (A-3), the values of δ^*, θ, and H for forced mixing are compared with those for pressure distributions C and D. We observe here a step-up $\Delta\delta_t^*$ and $\Delta\theta_t$, due to the drag of the device, which lifts the curves for δ^* and θ, aft of the 'device position', above the corresponding curves for C and D. Actually none of the curves are quite coincident ahead of this position due to the fact that the boundary layers have different pressure and skin-friction histories. The difference is, however, so small that the curves for the simple plow and distribution C may be regarded as coincident up to the occurrence of the step. The principal feature to be pointed out here is that the θ-curve for forced mixing lies above that for C by an almost constant percentage equal approximately to that given by $\Delta\theta_t/\theta_t$ in table 1. When only data for distributions B and C were available for comparison, this feature was observed to exist more or less in all cases. It was observed also that the H-curves generally lay close together. These tendencies were originally taken to mean that skin friction was playing a negligible role and that $\Delta\theta_t$ at the devices was affecting the subsequent development of the layer as would be expected from equation (2), namely, by a constant percentage as indicated above. There were, however, obvious discrepancies, and this led to the study of the boundary layer with the additional pressure distributions *D*, *E* and *F*. With the curves for *D* added to figure 10 it became clear at once that the foregoing percentage rule could not be universally applied.

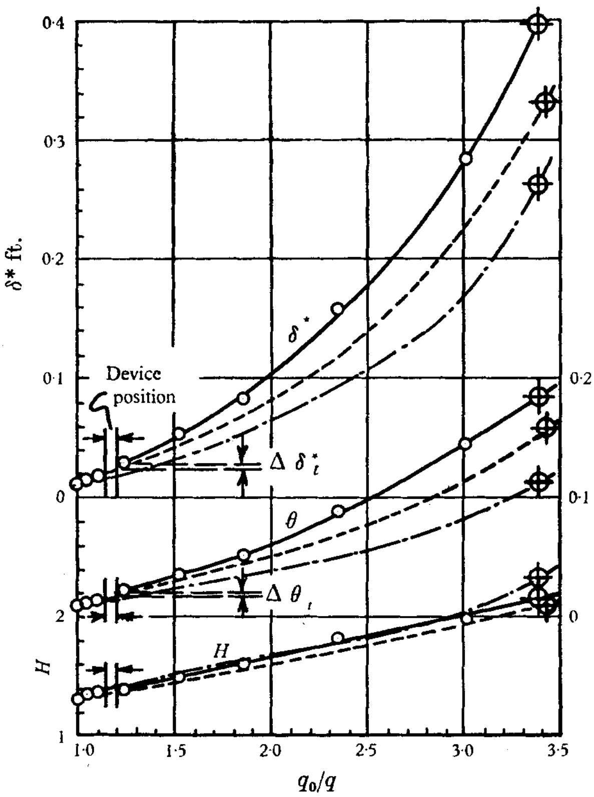

FIGURE 10. Comparison of the effect of forced mixing (conditions as given in table 1) with the effect of pressure distribution alone. δ^*, θ, H are means taken across the span. ⊕, Separation point; ○, simple plow (A-3); - - - -, pressure distribution *C*; —·—, pressure distribution *D*.

It thus became evident that skin friction must still be important and that the end result achieved with a pressure gradient would depend significantly on the shape of the pressure distribution. This raised the very troublesome question about how now to draw a meaningful comparison. It was concluded that a logical scheme would be to compare cases where pressure distributions were similar. Accordingly, distributions *E* and *F* were produced in an attempt to duplicate with a small gradient the nearly linear type of pressure increase existing when forced mixing was applied. Distribution *F* of figure 3 represents the maximum recovery obtainable with a straight-line gradient in the length available, but this was still not sufficient to match the recovery obtained with device (A-3). Distribution E was planned to match the recovery obtained with the triangular plow (E-2) and the shielded sink (K), making possible the comparisons shown in figure 11. Here again we see no significant differences until the occurrence of the steps $\Delta\delta_t^*$ and $\Delta\theta_t$. Inspection of the θ-curves reveals that the approximate percentage rule has now reappeared. In table 1 $\Delta\theta_t/\theta_t$ for (E-2) is given as 28·9 %; at separation the increase above the dashed curve is 27·2 %. Again in table 1 $\Delta\theta_t/\theta_t$ for (K) is $-17{\cdot}6$ %, while at separation the reduction is $-19{\cdot}4$ %. The rule was previously indicated for δ^* also, but the evidence for it is lacking in figure 11.

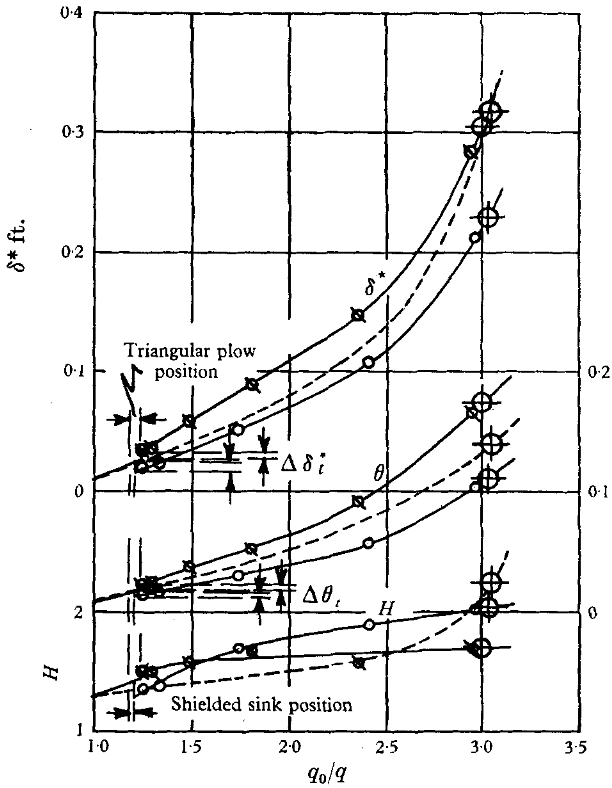

FIGURE 11. Comparison of the effect of forced mixing (conditions as given in table 1) with the effect of reduced pressure gradient. δ^*, θ, H are means taken across the span. ⊕, Separation point; ⊗, triangular plow (E-2); ○, shielded sink (K); – – –, pressure distribution E.

The original purpose of the sink was to provide one case with a negative $\Delta\theta_t$ with which to test the percentage rule. The above numbers indicate close agreement, closer in fact than we would have reason to expect. A firm basis for such a rule exists only when H is always the same function of q_0/q and the skin-friction term in equation (1) is negligible. The approximate conformity to the rule, which has run generally through the cases studied, indicates that the conditions are partially satisfied or that the effect of violations are mitigated by the occurrence of the same deviations in both systems. Evidently the friction avoided by the reduction in wall length with forced mixing is replaced by the greater intensity of mixing. While skin friction could not be determined from the distorted velocity profiles existing downstream from mixing devices and therefore is not known, we would expect intuitively that it would be increased considerably where high velocities occur near the wall as illustrated in figure 8. Apparently, however, forced mixing does not overemphasize the skin friction on the average, for if it did, the results for the shielded sink with negative $\Delta\theta_t$ would not conform to the rule. The rule can obviously be circumvented by emphasizing the reduction in skin friction in one case and not in another, as, for example, in comparing the performance of the simple plow and pressure distribution D in figure 10.

Mixing devices differ considerably in their effectiveness, as shown by table 1, but they all affect the mechanics of flow to higher pressures in about the same way. Furthermore, on a pressure-recovery basis the increased rate of mixing which they bring about is basically equivalent in its effect to decreased pressure gradient, as anticipated in § 2. Devices differ from one another in the magnitude and sign of $\Delta\delta_t^*$ and $\Delta\theta_t$, and by these quantities they differ in their effect from a decreased pressure gradient. Where the percentage rule can be applied to θ, the difference appears to be solely due to these quantities and presumably would disappear if $\Delta\delta_t^*$ and $\Delta\theta_t$ disappeared. When $\Delta\theta_t$ is positive, a penalty is imposed on forced mixing, not so much because of the force on the devices themselves, but because the resulting percentage momentum loss is magnified by being deposited in a developing boundary layer. When $\Delta\theta_t$ is negative there is a dividend of like kind from the same source. A negative $\Delta\theta_t$ must involve the removal of fluid having a momentum deficiency and involve some additional losses from fluid handling; hence not all of the dividend is clear profit.

The obvious advantage of forced mixing is the saving of wall length made possible by it for a given pressure recovery. For example, with distribution E we require a length of adverse pressure region equal to 16·6 ft. for a pressure-recovery coefficient of 0·67. Using either the triangular plow (E-2) or the shielded sink (K), we obtain the same coefficient in a length of only 6·8 ft.—a saving of 9·8 ft. If we make a similar comparison between distribution C and the simple plow (A-3), where now the pressure-recovery coefficient is 0·71, we find that the length is reduced from 16·1 to 7·3 ft.—a saving of 8·8 ft.

If we now lay aside forced mixing and concentrate on manipulating the pressure distribution into its optimum form (a form approached by distribution D) we may again obtain high pressure recovery in a relatively short distance. Comparing the length for D with that for C, we find a reduction from 16·1 to 9·8 ft.—a saving of 6·3 ft. By making the pressure gradient initially high and then progressively relaxing it just enough to avoid separation we apparently make the useful work load a maximun at all points and in so doing exploit the mixing capabilities of the prevailing turbulence to the fullest. There appears to be little definite information on what this does to the turbulence itself. By observing the action of tufts Stratford (1959) concludes that the level of turbulence increases near the wall for the condition of incipient separation. The measurements of Ruetenik & Corrsin (1955), pertaining to equilibrium flow in a diffuser of 1-degree half angle, show a turbulent kinetic energy 3·1 times that in a parallel channel. Clauser (1954, 1956), who proposed a concept of equi librium boundary layers in adverse pressure gradients and made a study of such layers, finds the eddy viscosity, which is effectively a constant for the outer 80–90% of the layer, to be given by $KU_1\delta^*$, where K is a constant equal to 0·018, independent of pressure gradient and equally applicable to constant pressure flow. This denotes something more than the usual increase of eddy viscosity with thickening of the layer; it denotes an increase caused as well by a change in the form of the profile. There has evidently been an increase in mixing capability brought about by rearrangement of the flow in the y-direction, the 'device' in this case being an adverse pressure gradient applied so as to maintain equilibrium. The only known confirmation of this from turbulence measurements comes from the work of Ruetenik & Corrsin, and this for a condition of small pressure gradient. More turbulence measurements in equilibrium flows would appear to be desirable. We point out in this connexion that flows with zero skin friction represent the high-gradient extreme of an equilibrium flow.

It is not clear how much this depends on an equilibrium condition. Hot-wire data such as those given by Schubauer & Klebanoff (1951), Newman (1951), Sandborn & Slogar (1955), and Robertson & Calehuff (1957), pertaining to non-equilibrium conditions, suggest that there is no marked effect of an adverse pressure gradient on the turbulence, other than to change the distribution across the layer. The principal effect on the mixing is therefore not through a change in rate but through a decrease in the mean velocity, thus increasing the stirring relative to downstream movement. Kline (1958) has observed streaks of backflow even in mild pressure gradients in diffusers, and he infers increasing mixing from this source. However, it is not evident from hot-wire results that there is a significant increase in absolute level of turbulence for the general case of non-equilibrium flow such as that implied for equilibrium flow.

Forced mixing, in the sense of this investigation, obtains rearrangement by involving the third dimension, z, and hence will always increase any pre-existing level of mixing. Thus it would be of considerable interest to apply it under the special conditions just discussed. This would mean, for example, applying forced mixing under conditions such that δ^*, θ, and H could properly be compared with the values in figure 10 for distribution D, or with cases where separation was avoided altogether by a progressive decrease in pressure gradient, allowing a little or a substantial margin as desired. Spatial irregularities which characterize forced mixing may prove to be troublesome in such cases. In application therefore there are certain pitfalls to be recognized. In practice, moreover, it may not be advantageous, and indeed not always possible, to give the pressure curve a special shape for the benefit of the boundary layer.

7. Conclusions

The major conclusions from this investigation are:

1. Forced mixing has basically the same effect on the boundary layer as a general reduction in pressure gradient.

2. Differences in δ^* and θ for forced mixing on the one hand and reduced

pressure gradient on the other arise from increments or decrements in these quantities introduced at the mixing station. These are magnified in the course of development of the layer. In the case of θ, an approximately constant percentage increase or decrease is maintained throughout the subsequent course of the layer. This rule does not apply unless the more gradual pressure rise has essentially the same form as that of the steep rise to which forced mixing was applied.

3. The mechanics of mixing is about the same for all of the devices tried, as well as for those described in the literature, namely, an induction into the boundary layer of currents of higher velocity usually accompanied by streamwise vortices. Devices differ in the manner of accomplishing this and in their effectiveness and drag penalty.

4. Since the mixing potentialities of natural turbulence are best exploited by a pressure distribution with an initially steep and progressively decreasing gradient, it would be of interest to study forced mixing for this condition. Of interest also are those cases where the gradients are regulated to avoid the actual occurrence of separation and yet are kept high enough to reduce skin friction to a negligible magnitude.

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